Exercise 76

Suppose the curve $y = x^4 + ax^3 + bx^2 + cx + d$ has a tangent line when x = 0 with equation y = 2x + 1 and a tangent line when x = 1 with equation y = 2 - 3x. Find the values of a, b, c, and d.

Solution

Take the derivative of the equation for the curve.

$$y' = \frac{d}{dx}(x^4 + ax^3 + bx^2 + cx + d)$$

= $\frac{d}{dx}(x^4) + \frac{d}{dx}(ax^3) + \frac{d}{dx}(bx^2) + \frac{d}{dx}(cx) + \frac{d}{dx}(d)$
= $\frac{d}{dx}(x^4) + a\frac{d}{dx}(x^3) + b\frac{d}{dx}(x^2) + c\frac{d}{dx}(x) + \frac{d}{dx}(d)$
= $(4x^3) + a(3x^2) + b(2x) + c(1) + (0)$
= $4x^3 + 3ax^2 + 2bx + c$

Use the fact that at x = 0, the slope of the curve is 2.

$$y'(0) = c = 2$$
 (1)

Use the fact that at x = 1, the slope of the curve is -3.

$$y'(1) = 4 + 3a + 2b + c = -3 \tag{2}$$

Also, use the fact that at x = 0, y = 1.

$$y(0) = (0)^4 + a(0)^3 + b(0)^2 + c(0) + d = d = 1$$
(3)

Also, use the fact that at x = 1, y = -1.

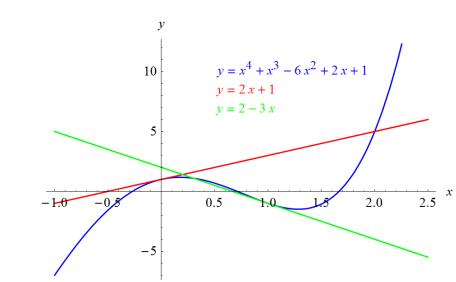
$$y(1) = (1)^4 + a(1)^3 + b(1)^2 + c(1) + d = 1 + a + b + c + d = -1$$
(4)

Solve equations (1), (2), (3), and (4) for a, b, c, and d.

 $a = 1 \qquad b = -6 \qquad c = 2 \qquad d = 1$

Therefore, the curve is

$$y = x^4 + x^3 - 6x^2 + 2x + 1.$$



Below is a graph of the curve with the two tangent lines at x = 0 and x = 1.